Design Sensitivity Analysis for Shape Optimization of Nonlinear Magnetostatic Systems

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The paper discusses the sensitivity analysis for the shape optimization of a non linear magnetostatic 2D/3D system, evaluated both by direct and adjoint approaches. The calculations are based on the material derivative and lie derivative concept of continuum mechanics. The resulting sensitivity formula can be expressed as a volume integral or as a boundary integral along the interface where shape modification occurs. A method for the calculation of the design velocity field and mesh updating scheme is developed as well. The accuracy of the methodology is analysed on an inductor system, leading to the conclusion that the volume integration technique should be preferred. All methods are freely available for further testing in the open source environment GetDP/Gmsh.

***Index Terms*—Design sensitivity analysis, adjoint variable method, direct method, continuum approach, magnetostatic system, shape optimization, lie derivative**

I. Introduction

Design sensitivity analysis is the computation of the derivative of a functional and the state variables of a system with respect to the design variables. To this end, two main approaches have emerged: the discretized approach and the continuum approach. The former consists in directly deriving the discretized equations of the system, while the latter first differentiates the continuous (partial differential) equations governing the system, before being discretized [1], [2], [3], [4]. In the case of magnetostatic systems, the continuum approach leads to a set of continuum sensitivity equations that can be solved numerically using the same discretization scheme as for the original problem. This approach is the most general, and can be easily implemented in any existing finite element code such as GetDP/Gmsh [5], [6]. In this paper we present both the direct and the adjoint continuum sensitivity analysis for a general nonlinear 2D and 3D magnetostatic system, taking into account the potential discontinuity of the state variable across bimaterial boundaries.

II. Formulation of the Magnetostatic Problem

Let us consider a magnetostatic problem modeled thanks to Ampère's equation (1) in a bounded domain $\Omega = \Omega_1 \cup \Omega_2$ with boundary $\partial\Omega = \Gamma$. Domains Ω_1 and Ω_2 are characterized by their own reluctivity ν and separated by an interface boundary γ undergoing shape modification as illustrated in Fig. 1. The system is excited by permanent magnets with a magnetization $\mathbf{M}$ and/or inductors with a current density $\mathbf{J}$, which can be located anywhere in the domain.

$$\mathbf{curl}\,(\nu\ \mathbf{curl}\,\mathbf{A}) = \mathbf{J} + \mathbf{curl}\,(\mathbf{M}) \quad \text{in } \Omega, \tag{1}$$
$$\mathbf{A} = 0 \quad \text{on } \Gamma. \tag{2}$$

Looking for the magnetic vector potential in an appropriate function space Z_A and using test functions in the same space, the classical weak formation reads

$$a\left(\mathbf{A}, \bar{\mathbf{A}}\right) = l\left(\bar{\mathbf{A}}\right),\ \forall \bar{\mathbf{A}} \in Z_A, \tag{3}$$

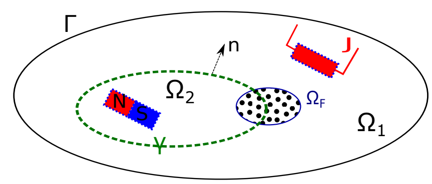

Fig. 1. Considered domain for the shape optimization problem.

with $a\left(\mathbf{A}, \bar{\mathbf{A}}\right) := \int_\Omega \nu\ \ \mathbf{curl}\,\mathbf{A} \cdot \mathbf{curl}\,\bar{\mathbf{A}}\, d\Omega$ and $l\left(\bar{\mathbf{A}}\right) := \int_\Omega \left(\mathbf{J} + \mathbf{curl}\,(\mathbf{M})\right) \cdot \bar{\mathbf{A}}\, d\Omega$.

III. Design Sensitivity Analysis

A. Material Derivative

Consider a continuous medium Ω, as shown in Fig. 1, where the shape of the interface boundary γ is controlled by a design variable τ. A velocity field $\mathbf{V}$ which associates τ to the movement of material points is applied to each material point of Ω with coordinates $\mathbf{x}$. Thus, a perturbed point $\mathbf{x}^\tau$ for a given geometry perturbation τ is obtained with:

$$\mathbf{x}^\tau = \mathbf{x} + \tau \mathbf{V}(\mathbf{x}, \tau). \tag{4}$$

Using differential forms proper to differential geometry [7] and (4), a material derivative can be obtained for the 1-form state variable $\mathbf{A}$:

$$\mathrm{d}\mathbf{A}/\,\mathrm{d}\tau := \dot{\mathbf{A}} + \mathbf{grad}\,\mathbf{V} \cdot \mathbf{A}, \tag{5}$$

where $\dot{(.)} \equiv (\partial_\tau + \mathbf{V} \cdot \mathbf{grad}\,)\,(.)$.

B. Sensitivity Formula Derivation

The aim is to find the derivative $\mathrm{d}F/\,\mathrm{d}\tau$ of a function $F = \int_{\Omega_F} f(\mathbf{A}(\tau), \tau)\,\mathrm{d}\Omega$ or $F = f(\mathbf{A}(\tau), \tau)$ that represents any performance measure or constraint in the optimization problem. Two different approaches can be used to compute $\mathrm{d}F/\,\mathrm{d}\tau$:

- The *direct differentiation approach* evaluates the dependence of $\mathbf{A}$ with respect to τ using the derivative of the weak formulation:

$$a\left(\mathrm{d}\mathbf{A}/\,\mathrm{d}\tau, \bar{\mathbf{A}}\right) = l'\left(\bar{\mathbf{A}}\right) - a'\left(\mathbf{A}, \bar{\mathbf{A}}\right),\ \forall \bar{\mathbf{A}} \in Z_A. \quad (6)$$

- The adjoint method uses a Lagrange multiplier λ as the solution of the system in (7), in order to avoid the evaluation of the response sensitivity $\mathrm{d}\mathbf{A}/\,\mathrm{d}\tau$. The solution is then used to retrieve $\mathrm{d}F/\,\mathrm{d}\tau$:

$$a\left(\lambda, \bar{\lambda}\right) = \left(\partial_{\mathbf{A}} F\right) \cdot \bar{\lambda},\ \forall \bar{\lambda} \in Z_\lambda. \quad (7)$$

The complete developments will be given in the full paper, highlighting two alternatives for the calculation of the terms, either using boundary integrals or using volume integration terms.

C. Velocity Field Computation

The velocity field is defined uniquely on the boundary $\partial\Omega$ ($\mathbf{V}_{\partial\mathbf{\Omega}}$), and it is arbitrary in the interior of the domain. At the discrete level, we determine $\mathbf{V}_{\partial\mathbf{\Omega}}$ geometrically: the mesh nodes are relocalized on the surface after a finite perturbation of the boundaries, based on the underlying parameterization of the geometrical (CAD) model. The velocity is then extended to the whole domain by considering two methods:

- The boundary layer extension method: from a known velocity field on the boundary nodes, the values are extended to the interior nodes located on the adjacent layer by using a linear interpolation (see Fig. 2). This method is very efficient, is very natural in a finite element context.
- The method of Laplacian smoothing (LS) [8] retrieves the inner nodes velocity field by solving a laplacian problem with boundary velocity field as a Dirichlet boundary condition. The result is illustrated in Fig.3.

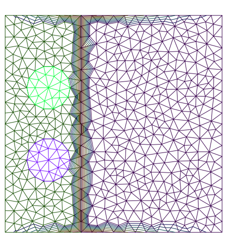

Fig. 2. Single layer velocity field.

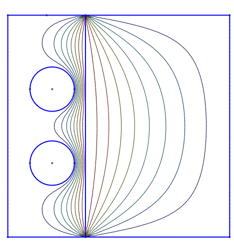

Fig. 3. LS velocity field.

IV. Application Example

The sensitivity analysis is validated with a rectangular domain subject to homogenous Dirichlet boundary conditions on Γ. A current density is imposed in a disk and a design function corresponding to the magnetic energy is defined on another disk. The difference of reluctivities between the rectangular regions is set to 1000. The velocity field is propagated to one layer. The corresponding magnetic potential vector and the adjoint variable are represented on Fig. 4 and Fig. 5, respectively. When the mesh is refined, the sensitivity computed using both the boundary integral and the volume integral converge

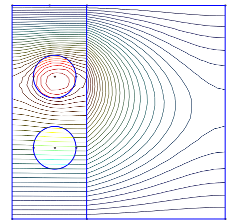

Fig. 4. z-component of the magnetic potential vector

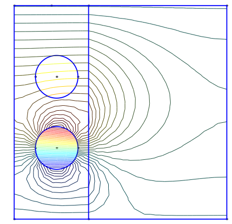

Fig. 5. z-component of the adjoint variable

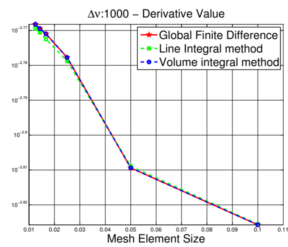

Fig. 6. Derivative vs mesh characteristic length

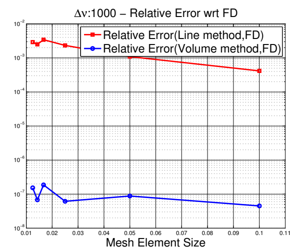

Fig. 7. Relative error of derivative vs mesh characteristic length

to the global finite difference solution (see Fig. 6). However, the relative error related to the line integration method is larger (see Fig. 7), suggesting that volume integration method should be preferred.

Acknowledgements

This work was supported in part by the Walloon Region of Belgium under grant RW-1217703 (WBGreen FEDO).

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